Rational Exponents

(exponents that are fractions)

In general, $a^{\frac{1}{n}} = \sqrt[n]{a}$ for all $a \geq 0$

$\sqrt[n]{a}$ is called a radical and means the nth root of a.

$2^{\frac{1}{5}}$ ⟷ $\sqrt[5]{2}$

Exponential Form — Radical Form

Commonly Calculated Roots/Radicals

$\sqrt{m} = m^{\frac{1}{2}}$ = the square root of m

$\sqrt[3]{m} = m^{\frac{1}{3}}$ = the cube root of m

$\sqrt[4]{m} = m^{\frac{1}{4}}$ = the fourth root of m

$\sqrt[5]{m} = m^{\frac{1}{5}}$ = the fifth root of m

Example Evaluate if possible.

a. $\sqrt[3]{-8}$

b. $\sqrt[5]{243}$

c. $27^{\frac{1}{3}}$

d. $\left(\frac{1}{16}\right)^{\frac{1}{4}}$

e. $\sqrt{-16}$

Now, what do we do if the fraction in the exponent does not have 1 as the numerator? What if the fraction is, for example, ¾?

Consider the radical $16^{\frac{3}{4}}$. We can re-write this power two different ways:

Which one is easier to evaluate? Why?

In general, $a^{\frac{m}{n}} \rightarrow \sqrt[n]{a^m}$, $a^{\frac{m}{n}} \rightarrow \left(\sqrt[n]{a}\right)^m$

<u>Example</u> Evaluate the following.

a. $32^{\frac{3}{5}}$ b. $125^{\frac{2}{3}}$ c. $8^{-\frac{5}{3}}$