Finding Optimum Volume and Surface Area

Recall from your investigations in the last lesson:

1. For a fixed perimeter, the rectangle with the largest area has __________ dimensions (ie. It is a ___________).

   eg. If you have 20m of fencing, the largest area you can enclose is _____ m^2:

2. If one edge of your enclosure is bounded (say, by a building), then fencing is only required along 3 sides:

   In this case, the largest area you can enclose is _____ m^2.

These concepts can be extended to maximizing volume and minimizing surface area in 3 dimensions. Companies would use these concepts, for example, when designing packaging in order to save money (less packaging means cost savings).

For a square-based prism with fixed volume, the prism with the smallest surface area has ___________ dimensions (ie. It is a ______________).

Example A box, in the shape of a square-based prism, is to have a volume of 64 000 cm^3 and a minimum surface area. Determine the dimensions of the box.

For a cylinder with fixed volume, the cylinder with the smallest surface area has ___________ _________________________________.

Example a. Find the volume of a cylinder with radius 2 m and height 16 m.

b. Find the height and radius of a cylinder that has this same volume but minimum surface area.

If you are working with a fixed surface area (say, a fixed amount of packaging) and you are trying to maximize volume:

For a square-based prism with fixed surface area, the prism with the greatest volume has ___________ dimensions (ie. It is a ______________).

Example Suppose you have $1000cm^2$ of cardboard that is to be used to make a container to hold popcorn. Find the dimensions of the container (in the shape of a square-based prism) that will hold the most popcorn.

Summary: Optimizing Volume and Surface Area

- **The minimum surface area for a given volume of a rectangular prism** occurs when the height is equal to the side length of the base (a cube)
- **Also: The maximum volume of a rectangular prism for a given surface area** occurs when the height is equal to the side length of the base (a cube)
- **The minimum surface area for a given volume of a cylinder** occurs when the height is equal to the diameter