Chapter 6

Get Set, pages 103-104

1. a) base 2, exponent 3, 8
b) base –3, exponent 2, 9
c) base 3, exponent 2, –9
d) base 3, exponent –2, $\frac{1}{9}$

2. a) 3
b) 2
c) 4
d) 5

3. a) 6
b) 9
c) 11
d) 13

4. a) 5.5
b) 7.8
c) 9.0
d) 11.2

5. a) 5^5
b) $(-2)^8$
c) 7^2
d) 9^6

6. a) x^5
b) $4a^3x^5$
c) $\frac{1}{8a^{12}}$ or $0.125a^{-12}$
d) c^2b^2

7. a) 1
b) –1
c) 1
d) 1

8. a) $\frac{1}{5}$, 0.2
b) $\frac{1}{3^2}$, $\frac{1}{9}$

c) $\frac{1}{6^3}, \frac{1}{216}$

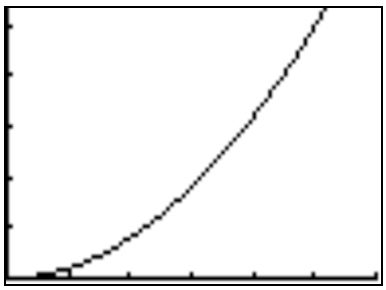
d) 3^2, 9

9. a)

x	y	First Differences	Second Differences	Ratio of Successive y-Values
0	0			
		3		3
1	3		6	
		9		4
2	12		6	
		15		2.25
3	27		6	
		21		
4	48		6	1.78
		27		1.56
5	75			

b) quadratic; the second differences are constant while the ratios of successive *y*-values are not constant

c)



Xmin = 0, Xmax = 6, Xscl =1, Ymin = 0, Ymax = 80,
Yscl = 15

No, the graph does not confirm my answer to part b) because it looks like an exponential curve. If the equation were graphed for negative values for *x*, then the graph would look like a parabola.

10. quadratic; the average of the second differences are constant and an equation for the relation is $A = 2w^2$

Section 6.1 Exponent Law, pages 105–107

Warm-Up

1. GCF: 7, LCM: 168

2. $4x^2 - 2x - 22y + 13$

3. $\frac{a^2y - 3}{5}$

4. 496 cm^2

5. No, one head and one tail is the theoretical probability with all things being equal. In the experiment, you could get two heads or two tails. However, if you repeat the experiment a great number of times, then the probability will average out to one head and one tail.

6. quadratic

7. B, 7.2

8. exponential

Practise

1. a) 12^{10}
b) 4^{-5}
c) 10^{2}
d) 5^{-6}

2. a) $\frac{1}{13}$
b) $\frac{1}{16}$
c) $\frac{25}{36}$
d) $\frac{1}{1323}$

3. a) $\frac{1}{4}$
b) $\frac{1}{5}$
c) $\frac{1}{256}$
d) $-\frac{1}{3}$

4. a) 5
b) $\frac{1}{32}$
c) 144 000
d) $\frac{49}{81}$
e) –72

5. a) $\frac{1}{729}$
b) 64
c) $\frac{1}{625}$
d) 16

6. a) $\frac{1}{256}$
b) $\frac{1}{11\,664}$
c) $\frac{1}{49}$
d) $\frac{1}{4}$
e) $\frac{1}{36}$

7. a) c^9
b) u^5
c) $\frac{1}{ts^{14}}$
d) m^3n^5
e) $\frac{1}{z^{21}}$
f) jk^3

8. a) 0.12
b) 10 011 291.5
c) 1 927 336.75
d) 390 625
e) 0.03
f) 0.04

9. a) 2
b) 2

10. a) $\frac{1}{36}$
b) $\frac{1}{36}$
11. $\frac{2}{729}$

Section 6.2 Rational Exponents, pages 108–110

Warm-Up

1. a) $\frac{9}{20}$

b) $\frac{5}{4}$

c) $\frac{11}{30}$

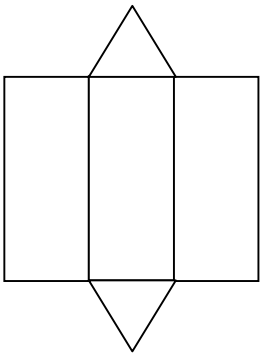
2. a) $\frac{r}{q}$

b) $\frac{a}{2b}$

c) $\frac{a^2}{b^2c}$

3. Graphs may vary. $y = 2^x$

4. Answers may vary. Sample answer:
triangular prism



5. 7

6. $y = 0.26x + 1.02$

7. Yes, the exponent is 3 which can be expressed as $\frac{3}{1}$.

8. $\frac{1}{u^8v^2}$

Practise

1. a) 12
b) 4
c) 10
d) 2

2. a) Not possible, there is no number that when multiplied by itself equals –16.
b) –7
c) Not possible, there is no number that when multiplied by itself four times equals –10 000.
d) –1

3. a) 3.11
b) 2.24
c) –2.51
d) 9.54

4. a) 2
b) 7
c) 6
d) 5

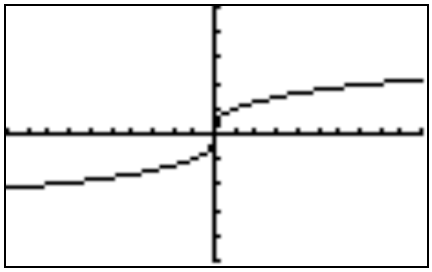
5. a) –4
b) Not possible, there is no number that when multiplied by itself equals –4.
c) –8
d) Not possible, there is no number that when multiplied by itself four times equals –16.

6. a) 4
b) 1000
c) 81
d) 128

7. a) 3.742
b) 2.262
c) 43.712
d) 2.081
e) 3.737
f) 10.079
g) 6.017
h) 3.684

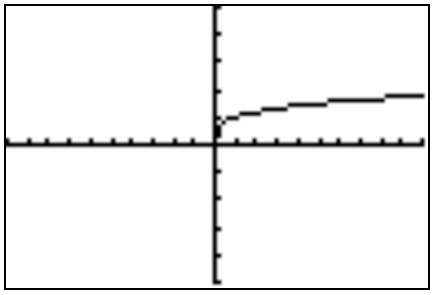
8. a) Answers may vary. Sample answer:
The curve looks like a horizontally stretched S, with turning point through the origin.
See part b) for graph.

b) Answers may vary. Sample answer:
My prediction was somewhat accurate.



Xmin = –10, Xmax = 10, Xscal = 1, Ymin = –5, Ymax = 5, Yscl = 1

c) The graph will look almost the same as the graph of $y = x^{\frac{1}{3}}$ except the graph does not exist for negative values of x.



Xmin = –10, Xmax = 10, Xscal = 1, Ymin = –5,
Ymax = 5, Yscl = 1

9. a) 24.1 cm^3

b), c) $V_{\text{cube A}} \doteq 6.1\ cm^3$, $V_{\text{cube B}} \doteq 17.2\ cm^3$
No, the volume of cube B is almost 3 times the volume of cube A.

10. a) 12.2%
b) 9.1%

11. $12a^6b^4$

Section 6.3 Represent Exponential Expressions, pages 111–113

Warm-Up

1. $\frac{101}{100}$, $\frac{104}{102}$, 1.04, $\frac{22}{21}$, 1.4

2. a) $\frac{1+a}{a^3}$

b) $\frac{m^4 + n^4}{m^2 n^3}$

c) $\frac{u^5 + v^2}{v^2 u}$

3. a), b) (2, 4)

4. a) $v = s^3$
b) 125 cm^3
c), d) No, it is 8 times as great (1000 cm^3 vs 125 cm^3).

5. No, since each event is not mutually exclusive, that is, it will not rain unless there are clouds, the probabilities should be multiplied.

6. $a = \sqrt{36 - b^2}$

7. $\sqrt[4]{15}$

8. a) 4
b) 27

Practise

1. a) 5^2
b) 5^6
c) 5^{-4}

2. a) 6^9
b) 6^4
c) 6^{-8}

3. a) 2^6
b) 6^{-4}
c) 4^6
d) 2^{12}
e) 2^0
f) 3^0

4. a) (2, 9)
b) The x-coordinate of the point of intersection is the solution to the equation.
c) $x = 2$

5. a), **b)** $x = -4$

6. a) $x = -3$

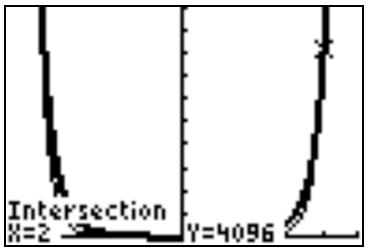
b) $a = -26$
c) $y = -24$
d) $m = -\frac{16}{7}$
e) $r = -8$
f) $b = 12$

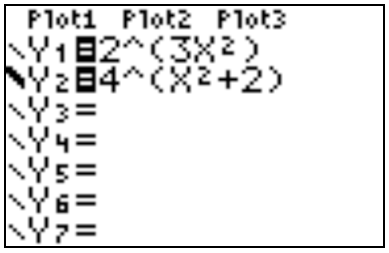
7. a) $w = 17$
b) $z = -2$

8. a) $A = 4096$, $B = 16\ 384$
b), d) $d = 6$
c), d) 262 144 bacteria
e) Bacteria A

9. a), b) $x = -32$
c) Answers may vary. Sample answer:
I prefer the algebraic method of solution. It is easier because I only have to solve for x. When I graph the equations and try to find the point of intersection, I have to figure out the proper window settings so that I can see the point. The range and domain have to be quite large in this case.

10. a) $2^{3x^2} = 2^{2(x^2+2)}$ or $2^{3x^2} = 2^{2x^2+4}$
b) $x = 2$ or -2
c), d) The graphs look like two parabolas superimposed onto each other, with intersection points at (–2, 4096) and (2, 4096).





Xmin = –2.5, Xmax = 2.5, Xscl = 0.5, Ymin = –100, Ymax = 5000, Yscl = 500

11. a) $2^x = 4^{x-2}$
b) The x-coordinate of the point of intersections of the two graphs represents the solution to the equation in part a). $x = 4$

Section 6.4 Tools and Strategies to Solve Equations Involving Exponents
Warm-Up, pages 114–116

1. a) 4.08
b) 4.85
c) 49.35

2. a) $x = -\frac{12}{y-5}$

b) $x = \frac{z}{5y+z^2}$

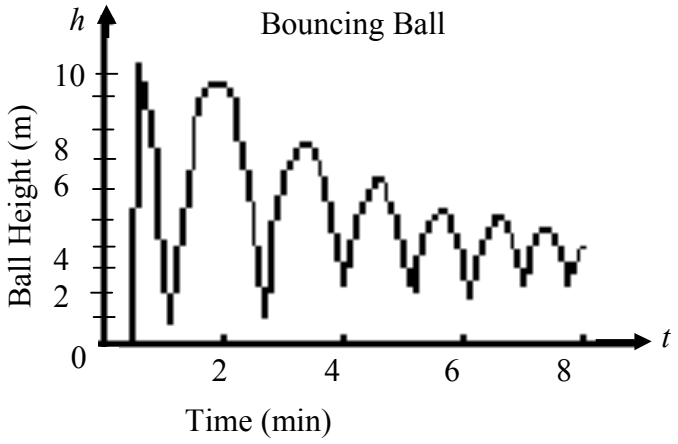
c) $x = \frac{m}{3a+4b^8}$

3. $x \doteq 210.04$

4. 30º, 150º, 150º

5. Answers may vary. Sample answer:
The question is misleading in that it will lead to response bias.

6. Answers may vary. Sample answers:
a) Sasha should see a series of connected downward parabolas but with decreasing maximums as time increases.



b) It is assumed that no external forces are acting on the ball other than gravity. A certain percent of energy is lost due to heat and noise with each bounce.

7. B, 3^x

8. a) $x = 1$
b) $y = 2$
c) $z = 17$

Practise

1. a) $m \doteq 2.29$

b) $c \doteq -3.42$

c) $r \doteq 3.83$

d) $h \doteq 4.25$

2. a) $r \doteq 5.99$

b) $r \doteq 1.78$

c) $r \doteq 1.34$

3. Answers may vary. Sample answers:

a) $x \doteq 3.5$; $2^3 = 8$ and $2^4 = 16$, 12 is halfway between 8 and 16, so the exponent could be halfway between 3 and 4 or 3.5.

b) $k \doteq 1.4$; $15^1 = 15$ and $15^2 = 225$, 100 is less than halfway between 15 and 225, so the exponent could be less than halfway between 1 and 2 or 1.4.

c) $y \doteq 2.3$; $5^2 = 25$ and $5^3 = 125$, 52 is less than halfway between 25 and 125, so the exponent could be less than halfway between 2 and 3 or 2.3.

d) $b \doteq 0.25$; $50^0 = 1$ and $50^1 = 50$, 2 is much less than halfway between 1 and 50, so the exponent could be much less than halfway between 0 and 1 or 0.25.

4. a) $x \doteq 3.585$

b) $k \doteq 1.701$

c) $y \doteq 2.455$

d) $b \doteq 0.177$

5. $x \doteq 2.8$

6. $x \doteq 6.2$

7. The error occurs in the fifth line. The fifth root of 400 hundred should be determined, not the square root.

$$\frac{2m^5}{3} = 400$$
$$2m^5 = 3(400)$$
$$m^5 = \frac{1200}{2}$$
$$m^5 = 600$$
$$\sqrt[5]{m} = \sqrt[5]{600}$$
$$m \doteq 3.59$$

8. a) $v = \frac{10}{3}\pi r^2$
b) 5.35 m

9. a) 4.4 years
b) 5.8 years
c) Paolo is incorrect. Option 2 means it will take longer to reach $1200.

10. a) 8 days
b) Colony 2

Section 6.5 Construct and Apply Exponential Models, pages 117– 120

Warm-Up

1. a) 10.51
b) 445.69
c) 0.11

2. A negative sign was introduced in the second line.
$$\frac{3r}{4^2} = 6$$
$$3r = (4^2)6$$
$$3r = 96$$
$$r = 32$$

3. D

4. cosine law

5. A

6. a) $A = \frac{1}{2}b^2$ or $A = \frac{1}{2}h^2$

b) $b > 0$, $h > 0$

7. Answers may vary. Sample answers:
a) The number of bacteria is increasing at an increasing rate, not a constant rate as in a linear model. So increases from one time period to the next are greater than in the previous time period.
b) the rate of increase and the initial number of bacteria in the colony

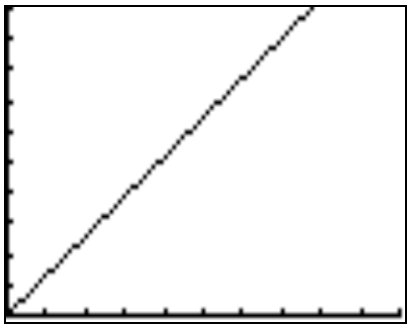
8. a) $x \doteq 2.26$

b) $y \doteq 1.54$

c) $h \doteq 1.29$

Practise

1. a)



Xmin = 0, Xmax = 10, Xscl = 1, Ymin = 0, Ymax = 10, Yscl = 1
b) Linear; for the equation given, first differences are constant. The graph confirms this..
c) $537.09
d) $446.85

2. a) the initial amount of the investment, $400
b), c)

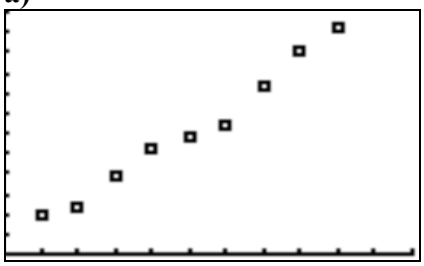
n	*A*	First Differences	Second Differences	Ratio of Successive *A* Values
0	400			
		17.20		
1	417.20		0.74	1.043
		17.94		
2	435.14		0.77	1.043
		18.71		
3	453.85		0.81	1.043
		19.52		
4	473.37		0.83	1.043
		20.35		
5	493.72			1.043

d) Exponential; the ratio of successive A-values is constant.
e) i) $493.72

ii) $928.42
f) 26.1 years

3. a) The interest rate is 4.3% for both relations.
b) Question 1 relation is a linear relation, question 2 relation is an exponential relation.

4. Answers may vary. Sample answers:
a)

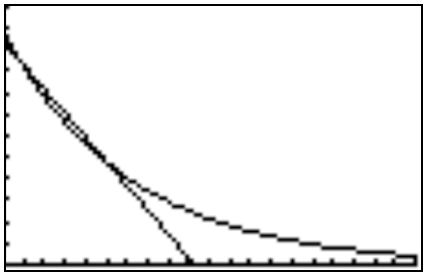


Xmin = 0, Xmax = 11, Xscl =1, Ymin = 0, Ymax = 600, Yscl = 50

As time increases, the number of cars per 1000 people increases.

b) All of the models are good representatives. The best model is linear with $r \doteq 0.989\ 39$ or quadratic with $r \doteq 0.989\ 37$, then exponential with $r \doteq 0.982\ 55$.
c) The model does not include the years in between the end of each decade or the years after 2000. Energy supply, economic recession, and other factors may affect the number of cars per 1000 people.

5. a)

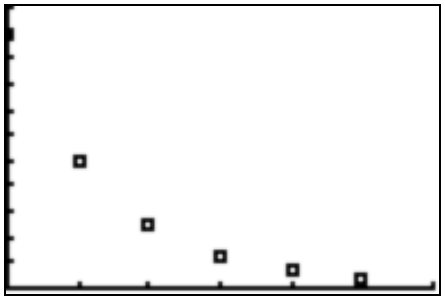


Xmin = 0, Xmax = 20, Xscl =1, Ymin = 0, Ymax = 3000, Yscl = 250
b) C3
c) XR; its resale value is $1018.31 while the resale value of the C3 is $920.

6.a)

Half-Life	Amount Remaining (mg)	First Differences	Second Differences	Ratios
0	1000			
		500		
1	500		250	
		250		$\frac{1}{2}$
2	250		125	
		125		$\frac{1}{2}$
3	125		62.5	
		62.5		$\frac{1}{2}$
4	62.5		31.25	
		31.25		$\frac{1}{2}$
5	31.25			

b) Exponential; the ratio of success mass values is constant.
c) Answers may vary. For example, 3.6 half-lives or 5.4×10^{17} years
d)



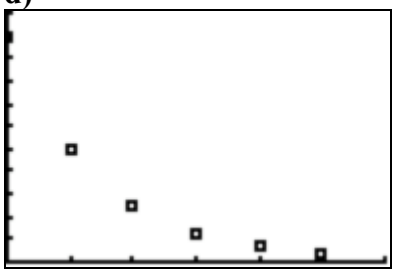
Xmin = 0, Xmax = 6, Xscl =1, Ymin = 0, Ymax = 1100, Yscl = 100
Yes, the graph confirms the trend.
e) $y = 1000(0.5)^x$
f) approximately 3.3 half-lives or 4.95×10^{17} years

7. a), b)

Half Life (periods)	Mass Remaining (mg)	First Differences	Second Differences	Ratio of Successive Mass Values
0	100			
		50		
1	50		25	
		25		$\frac{1}{2}$
2	25		12.5	
		12.5		$\frac{1}{2}$
3	12.5		6.25	
		6.25		$\frac{1}{2}$
4	6.25		3.125	
		3.125		$\frac{1}{2}$
5	3.125			

c) Exponential; the ratio of success mass values is constant.

d)



Xmin = 0, Xmax = 6, Xscl =1, Ymin = 0, Ymax = 1100, Yscl = 100

Yes, the graph confirms the trend.

e) $y = 100(0.5)^x$

f) approximately 4.3 half-lives or 8.6 years

Chapter 6 Review, pages 121–122

1. a) 216

b) $\frac{1}{2401}$

c) 100 000

d) $\frac{1}{2\,985\,984}$

e) $\frac{1}{36}$

f) 16

2. a) z

b) r^9

c) $\frac{1}{m^4}$

d) $\frac{1}{a^8b^4}$

e) $\frac{1}{2s^4}$

f) $\frac{1}{c^8d^{12}}$

3. $\frac{16}{81}$

4. a) 0.006

b) 60.84

c) 0.002

5. a) Not possible; there is no number that when multiplied by itself will equal –121.

b) 5
c) –4
d) 10
e) Not possible; there is no number that when multiplied by itself six times will equal –1 000 000.
f) 5

6. a) Not possible; there is no number that when multiplied by itself four times will equal –1.
b) 2
c) –2
d) 4
e) 243
f) Not possible, there is no number that when multiplied by itself two times will equal –16.

7. a) 5^4
b) 5^{-9}
c) 5^0

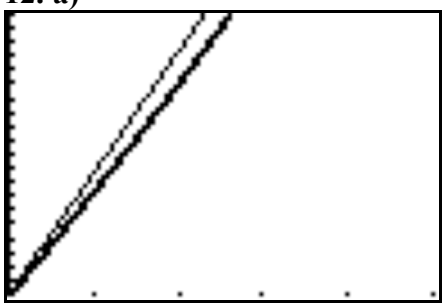
8. a) $x = -4$
b) $x = -2$
c) $y = 30$
d) $s = 5$
e) $x = 3$
f) $z = -11$

9. a) i) 200 insects
ii) approximately 283 insects
b) i) approximately 4.6 years
ii) approximately 6.6 years

10. a) the first video
b) the second video
c) Yes, approximately 39433 viewers after approximately 3 days (2.7, 39432.7).

11. Answers may vary. Sample answers:
a) Yes; food supply, presence of predators, environmental hazards, and other factors will affect population growth. So the population will not continue to grow at an increasing rate.
b) No; the popularity of the videos will wane as time goes on and new videos become available. The exponential models will not be maintained over time.

12. a)



Xmin = 0, Xmax = 5, Xscl = 1, Ymin = 700, Ymax = 800, Yscl = 5

The graph for Option A is a line.
The graph for Option B is an exponential curve when the equation is graphed for a longer time period.
b) i) Option A
ii) Option B