Chapter 5 Answers

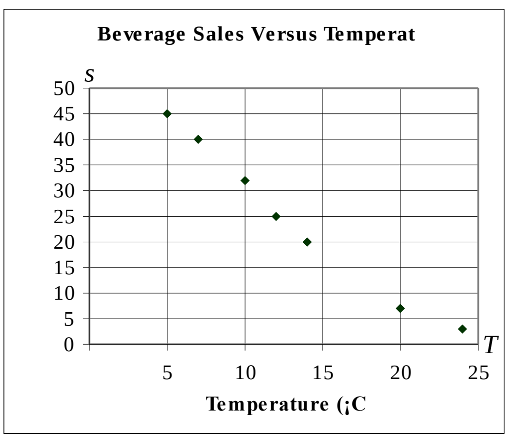
Get Set, pages 81-82

1. a) none of these **b)** always decreasing
c) always increasing **d)** constant

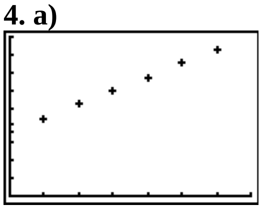
2.

x	*y*	First Differences	Second Differences
–3	–15		
		3	
–2	–12		4
		7	
–1	–5		–6
		1	
0	–4		0
		1	
1	–3		6
		7	
2	4		–4
		3	
3	7		

3.



4. a)



Xmin = 0, Xmax = 7, Xscl = 1, Ymin = 0, Ymax = 220, Yscl = 22
b) $y = 0.78x + 90.1$

5. a) 94.9 mL **b)** 65.6 mL **c)** after approximately 6.5 h
6. a) $6470 **b)** $4574.85

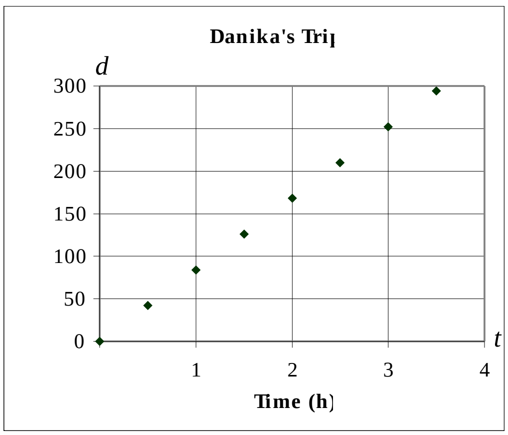
5.1 Linear Models, pages 83–85

Warm-Up

1. a) 7 **b)** 12

2. a) $5x - 11$ **b)** $4x^2 - 12x + 6y$
3. (–2, –5)
4. 10.5 cm
5. $\frac{2}{3}$
6. 893 cm^2
7. right isosceles triangle
8. a) 1000 **b)** 64 000

Practise
1. a) linear **b)** non-linear **c)** non-linear **d)** linear
2. a) decreasing **d)** increasing
3. a) \$9.65/h **b)** \$106.15
4. a)



The points form a straight line with a positive slope.
b) constant **c)** 84 km/h
5. Answers may vary. For example:
a) Gross earnings compared to hours worked.
b) The volume of water in a bathtub (drain plugged, water off) compared to time.
c) Fuel remaining compared to time spent travelling.
6. a) As the number of days increases, the area left to be harvested decreases by a constant amount.
b) approximately 1100 acres **c)** 92 acres **d)** acres/day
e) Constant. Walter has 92 fewer acres to harvest after each day.
f) approximately 16 days

5.2 Quadratic Models, pages 86–88
Warm-Up
1. a) 22 **b)** –6
2. $6x^2 - 3x + 26$
3. a) quadratic **b)** linear
4. 328 cm^2
5. $\frac{3}{26}$
6. Joe: $E = 10.25h$; Adriana: $E = 150 + 4.65h$
7. D
8. The lines are parallel. The first differences are 2 for both lines.

Practise

1. a) quadratic **b)** linear

2. a) quadratic

x	y	First Differences	Second Differences
–1	16		
		–2	
0	14		**–4**
		–6	
1	8		**–4**
		–10	
2	–2		**–4**
		–14	
3	–16		

b) neither

x	y	First Differences	Second Differences
0	1		
		1	
1	2		**1**
		2	
2	4		**2**
		4	
3	8		**4**
		8	
4	16		

c) quadratic

x	y	First Differences	Second Differences
–2	3		
		–3	
–1	0		**1**
		–2	
0	–2		**1**
		–1	
1	–3		**1**
		0	
2	–3		

d) linear

x	y	First Differences	Second Differences
–4	–1		
		3	
0	2		**0**
		3	
4	5		**0**
		3	
8	8		**0**
		3	
12	11		

3. a) $y = –2x^2 – 4x + 14$ **c)** $y = 0.5x^2 – 1.5x – 2$ **d)** $y = 0.75x + 2$

4. No. The x-values do not change by a constant amount.

5. a) As the ticket price increases, revenue increases to a maximum then decreases.

b) i) approximately \$20 500 **ii)** approximately \$19 100

c) Increasing for ticket prices less than \$15 and decreasing for ticket prices greater than \$15.

6. a) A quadratic model fits the data best.

Edge Length (cm)	Surface Area (m^2)	First Differences	Second Differences
15	0.135		
		0.405	
30	0.540		**0.27**
		0.675	
45	1.215		**0.27**
		0.945	
60	2.160		**0.27**
		1.215	
75	3.375		**0.27**
		1.485	
90	4.860		

b) increasing **c)** $y = 0.0006x^2$ **d)** 6.615 cm^2

5.3 Exponential Models, pages 89–91

Warm-Up

1. a) –27 **b)** 10 000 **c)** –25 **d)** 16

2. a) $2(x^2 + 3y)$ **b)** $7x(x^2 – 2x + 3)$

3. a) x-intercepts: (4, 0), (8, 0); y-intercept: (0, 32) **b)** (6, –4); minimum:

4. 2 cm
5. mean: 33.06; median: 30; mode: 24, 30
6. 6 kg of the \$16/kg tea; 4 kg of the \$20/kg tea
7. the axis of symmetry
8. $y = -\frac{1}{2}(x-4)^2 + 2$

Practise
1. a) 512 grains of rice
b) The number of grains doubles each time, which is like repeatedly multiplying by two.
2. a) Yes. Each term is ten times the previous term.
b) No. Each term is three more than the previous term.
c) Yes. Each term is double the previous term.
3. a) exponential

x	*y*	First Differences	Second Differences	Ratios
–3	–64	**48**	**–36**	$\frac{1}{4}$
–2	–16	**12**	**–9**	$\frac{1}{4}$
–1	–4	**3**	$-\frac{9}{4}$	$\frac{1}{4}$
0	–1	$\frac{3}{4}$	$-\frac{9}{16}$	$\frac{1}{4}$
1	$-\frac{1}{4}$	$\frac{3}{16}$		$\frac{1}{4}$
2	$-\frac{1}{16}$			

b) linear

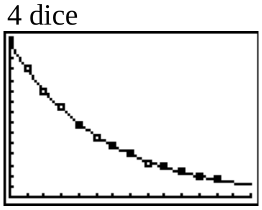
x	*y*	First Differences	Second Differences	Ratios
–2	6.0	**–1.1**	**0**	**0.8167**
–1	4.9	**–1.1**	**0**	**0.7755**
0	3.8	**–1.1**	**0**	**0.7105**
1	2.7	**–1.1**	**0**	**0.5926**
2	1.6	**–1.1**		**0.3125**
3	0.5			

4. a) As time increases, the value of the vehicle decreases at a decreasing rate.
b) approximately \$3600 **c)** dollars/year
d) Decreasing. The graph becomes less steep.

5. a)

Toss	Dice Remaining	First Differences	Second Differences	Ratios
0	72			
1	61	**–11**		**0.8472**
2	50	**–11**	**0**	**0.8197**
3	42	**–8**	**3**	**0.8400**
4	34	**–8**	**0**	**0.8095**
5	28	**–6**	**2**	**0.8235**
6	24	**–4**	**2**	**0.8571**
7	20	**–4**	**0**	**0.8333**
8	16	**–4**	**0**	**0.8000**
9	14	**–2**	**2**	**0.8750**
10	12	**–3**	**–1**	**0.8571**
11	10	**–2**	**1**	**0.8333**
12	8	**–2**	**0**	**0.8000**

b) Exponential. Ratios are all approximately 0.8.
c) $y = 71.59(0.8338)^x$
d) 4 dice

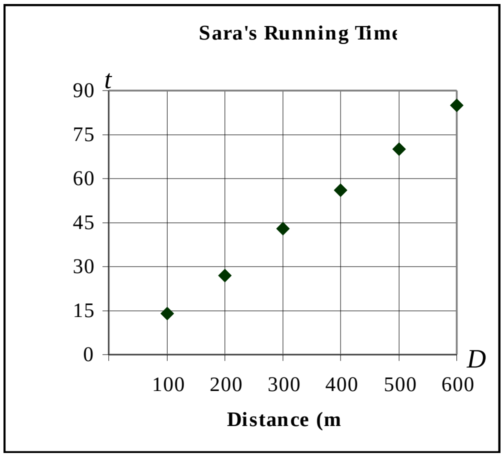


e)
Xmin = 0, Xmax = 14, Xscl = 1, Ymin = 0, Ymax = 75, Yscl = 5

5.4 Analyse Graphical Models, pages 92–95
Warm-Up

1. a) $\frac{17}{24}$ **b)** $\frac{3}{10}$ **c)** $\frac{1}{6}$
2. a) $(x – 7)(x + 5)$ **b)** $9(x + 1)(x + 2)$
3. $x = 1$
4. pentagon, octagon, decagon

5.



6. 80 km/h: 160 km; 100 km/h: 300 km
7. B
8. a) 13.81 s **b)** 600 g

Practise
1. a) C **b)** B **c)** A
2. C
3. a)

Year	Town X Population	Town Y Population
0	**90 000**	**4 000**
1	**115 000**	**8 000**
2	**140 000**	**16 000**
3	**165 000**	**32 000**
4	**190 000**	**64 000**
5	**215 000**	**128 000**

b) Town X: $P = 90\ 000 + 25\ 000t$; Town Y: $P = 4000(2)^x$
c) year 6
4. a)

Year	Option A Salary ($)	Option B Salary ($)
0	**48 000**	**45 000**
1	**49 000**	**46 125**
2	**50 000**	**47 278**
3	**51 000**	**48 460**
4	**52 000**	**49 672**
5	**53 000**	**50 913**
6	**54 000**	**52 186**
7	**55 000**	**53 491**

b) Option A. She will earn a higher salary with Option A until year 11, when the salary for Option B will be higher. Her total cumulative earnings for Option A will be greater than for Option B until year 21.

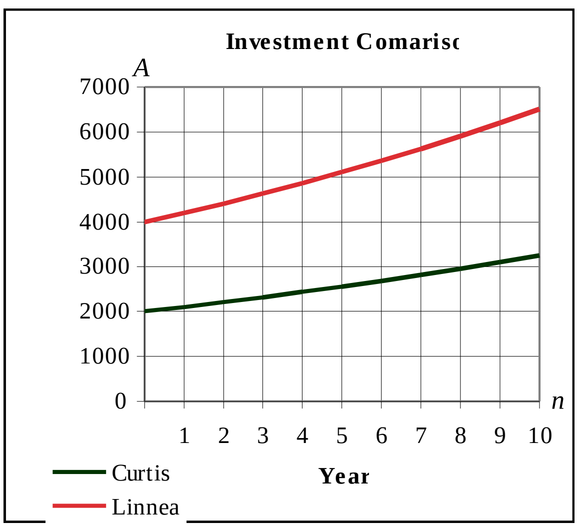
5. a)

Curtis				
Year	**Amount ($)**	**First Differences**	**Second Differences**	**Ratios**
0	**2000.00**			
		100.00		**1.05**
1	**2100.00**		**5.00**	
		105.00		**1.05**
2	**2205.00**		**5.25**	
		110.25		**1.05**
3	**2315.25**		**5.50**	
		115.75		**1.05**
4	**2431.00**		**5.81**	
		121.56		**1.05**
5	**2552.56**			

Linnea				
Year	**Amount ($)**	**First Differences**	**Second Differences**	**Ratios**
0	**4000.00**			
		200.00		**1.05**
1	**4200.00**		**10.00**	
		210.00		**1.05**
2	**4410.00**		**10.50**	
		220.50		**1.05**
3	**4630.50**		**11.00**	
		231.50		**1.05**
4	**4862.00**		**11.63**	
		243.13		**1.05**
5	**5105.13**			

b) dollars/year

c) Curtis: $A = 2000(1.05)^n$; Linnea: $A = 4000(1.05)^n$



d) The ratio increases. The amount of Linnea's investment is increasing at a faster rate.

6. Yes. Both accounts would increase at a constant rate of 1.05, so the ratio would remain constant.

5.5 Select a Mathematical Model, pages 96–99
Warm-Up

1. a) $\frac{41}{50}$ **b)** $\frac{7}{20}$ **c)** $\frac{9}{16}$

2. a) $(3x + 1)(3x - 1)$ **b)** $(2x + 1)^2$

3. a) $V = \frac{3}{2}A$ **b)** 21 V

4. 42°

5. a) 28 students **b)** 9 students

6. $h = -0.003d^2 + 0.5d$

7. vertex

8. after 10 years

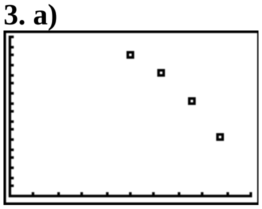
Practise

1. a) exponential **b)** exponential **c)** quadratic

2. a) Linear. The rate of change appears to be constant and positive.

b) No. There is not enough data.

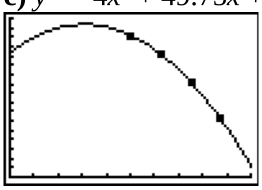
3. a)



Xmin = 0, Xmax = 20, Xscl = 2, Ymin = 0, Ymax = 850, Yscl = 50

b) Quadratic. The rate of change is increasing.

c) $y = -4x^2 + 49.75x + 662.90$



Xmin = 0, Xmax = 20, Xscl = 2, Ymin = 0, Ymax = 850, Yscl = 50

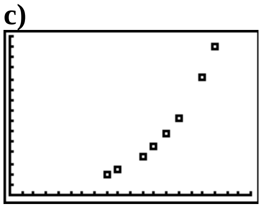
d) $x = 57.91$

4. a)

x	*y*	First Differences	Second Differences	Ratios
2.2	11.2	**2.8**	**0.6**	**1.25**
2.4	14.0	**3.4**	**0.9**	**1.24**
2.6	17.4	**4.3**		**1.25**
2.8	21.7			

b) Yes. Exponential. First and second differences are not constant but ratios are all approximately 1.25.

c)

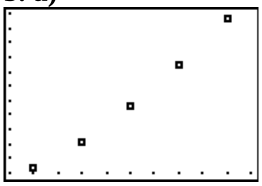


Xmin = 0, Xmax = 4, Xscl = 0.2, Ymin = 0, Ymax = 45, Yscl = 3

d) exponential

e) $y = (3)^x$

5. a)



Xmin = 0, Xmax = 10, Xscl = 1, Ymin = 40 000, Ymax = 95 000, Yscl = 5000

b) As the year increases, the value increases; the rate appears to be increasing

c) exponential

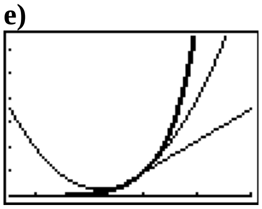
Year	Average Value ($)	First Differences	Second Differences	Ratios
1971	42 000			
		9 000		**1.2143**
1973	51 000		**3000**	
		12 000		**1.2353**
1975	63 000		**2000**	
		14 000		**1.2222**
1977	77 000		**2000**	
		16 000		**1.2078**
1979	93 000			

d) linear: $V = 6400t - 12\ 574\ 800$, $r^2 = 0.9888$

quadratic: $V = 285.71t^2 - 1\ 122\ 171t + 1\ 101\ 887\ 200$, $r^2 = 0.9999$

exponential: $V = 8.81 \times 10^{-82}(1.1053)^t$, $r^2 = 0.9997$

e)



Xmin = 0, Xmax = 90, Xscl = 15, Ymin = 0, Ymax = 400 000, Yscl = 5000

f) i) linear: –158 800; quadratic: 188 914; exponential: 1886

ii) linear: 289 200; quadratic: 636 914; exponential: 2 081 879

g) None of the models work from 1930 to 2020.

Chapter 5 Review, pages 100–102

1. a) A **b)** C, E **c)** B, D **d)** km/h

2. a)

x	y	First Differences	Second Differences
–1	–0.4		
		1.2	
1	0.8		**1.6**
		2.8	
3	3.6		**1.6**
		4.4	
5	8.0		**1.6**
		6	
7	14.0		**1.6**
		7.6	
9	21.6		

b) Quadratic. Second differences are constant.

3. a) approximately 375 m **b)** approximately 10 s **c)** m/s

d) Increasing from 0 s to 10 s and then decreasing.

4. a)

Year	Percent Remaining	First Differences	Second Differences	Ratios
0	100.0			
		–11.3		**0.8870**
5	88.7		**1.3**	
		–10.0		**0.8873**
10	78.7		**1.2**	
		–8.8		**0.8882**
15	69.9		**0.9**	
		–7.9		**0.8870**
20	62.0		**0.9**	
		–7.0		**0.8871**
25	55.0		**0.8**	
		–6.2		**0.8873**
30	48.8			

b) Exponential. Ratios are all approximately 0.887.

c) $y = 99.99(0.9764)^x$ **d)** approximately 29 years

5. a)

Time (years)	Amount in Account A ($)	Amount in Account B ($)
0	**4000.00**	**4000.00**
1	**4168.00**	**4152.00**
2	**4336.00**	**4309.80**
3	**4504.00**	**4473.50**
4	**4672.00**	**4643.50**
5	**4840.00**	**4820.00**
6	**5008.00**	**5003.16**
7	**5176.00**	**5193.28**

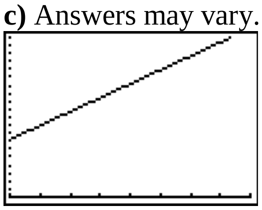
b) It depends on how long he is investing the money. If he is investing it for less than seven years, Account A will earn more interest. If he is investing it for seven years or more, Account B will earn more interest.

6. a)

Year	Average Amount ($)	First Differences	Second Differences	Ratios
2000	6214.00			
		434.98		**1.07**
2001	6648.98		**–0.04**	
		434.94		**1.07**
2002	7083.92		**0.03**	
		434.97		**1.06**
2003	7518.89		**0.04**	
		435.01		**1.06**
2004	7953.90			

b) Answers may vary. For example: Linear. First differences are approximately equal. Second differences are close to zero. The *r*-value for the linear model is closer to 1 than for the exponential or quadratic models.

c) Answers may vary. For example: $y = 434.97x - 863\ 728$



Xmin = 0, Xmax = 16, Xscl = 2, Ymin = 0, Ymax = 10 000, Yscl = 500

d) Answers may vary. For example:

i) $4039.14 **ii)** $10 563.71